460-2 International Economics

Problem Set 1

1. (Aguiar and Gopinath, 2007) Consider a small open economy receiving the stochastic endowment y_t, which is made of two components, a permanent component x_t and a temporary component z_t:

$$y_t = x_t + z_t,$$

where x_t follows the process

$$\Delta x_t = \rho_x \Delta x_{t-1} + \varepsilon_t,$$

z_t follows the process

$$z_t = \rho_z z_{t-1} + \eta_t,$$

and ε_t and η_t are i.i.d. shocks. The consumers have preferences

$$E_0 \sum \beta^t u(c_t)$$

with a quadratic utility function $u(c_t) = c_t - (1/2)\, c_t^2$. Consumer observe x_t and z_t when they are realized. The country has access to a one period, risk free bond, at the world interest rate r, which satisfies $\beta(1+r) = 1$.

(i) Setup the problem of the representative consumer in recursive form, using the appropriate state space. Derive the optimality conditions for c_t.

(ii) Let a_{t-1} denote the country's current holdings of international bonds and conjecture that optimal consumption takes the linear form

$$c_t = b_0 + b_1 x_t + b_2 x_{t-1} + b_3 z_t + b_4 a_{t-1}.$$

Derive the equilibrium values of the parameters b.

(iii) Show that consumption growth Δc_t is a stationary time series.

(iv) Derive the current account for the country as a moving average of past shocks. Show that CA_t is a stationary time series.

(v) Fix $\beta = 0.96$. Vary the value of $\sigma_\varepsilon^2 / \sigma_\eta^2$ and plot the following two statistics

$$Corr\left[CA_t, \Delta y_t\right]$$
$$Var\left[\Delta c_t\right] / Var\left[\Delta y_t\right]$$

Compare with Figure 4 in Aguiar and Gopinath (2007). Discuss.

2. (Kraay and Ventura, 2000) Consider an economy where consumers can invest in two assets: home capital and foreign capital. Total investment in the two assets is denoted by k_t, k_t^*. Home capital and foreign capital are risky with random linear returns A_t and A_t^*.

Consumers preferences are represented by

$$E\left[\sum_{t=0}^{\infty}\beta^t \ln c_t\right]$$

and the flow budget constraint is

$$k_{t+1} + k_{t+1}^* + c_t = A_t k_t + A_t^* k_t^*.$$

(Relative to what we saw in class there are no bonds here).

Assume that A_t and A_t^* follow Markov processes:

$$\begin{aligned} A_t &= (A_{t-1})^\rho \epsilon_t \\ A_t^* &= \left(A_{t-1}^*\right)^\rho \epsilon_t^* \end{aligned}$$

where ϵ_t and ϵ_t^* are i.i.d. log normal disturbances with mean 1. The variances of ϵ_t and ϵ_t^* are equal to σ^2.

Define wealth (with dividends) as

$$w_t = A_t k_t + A_t^* k_t^*$$

(i) Show that the optimal consumption rule is linear and equal to

$$c_t = (1-\beta)\, w_t$$

(ii) Show that the optimal portfolio share $\theta_t \equiv k_{t+1}/\left(k_{t+1} + k_{t+1}^*\right)$ is time varying and is given by the function

$$\theta_t = h\left(a_t\right)$$

where $a_t \equiv A_t/A_t^*$.

(iii) Show that $h\left(.\right)$ is a nondecreasing function.

(iv) Use a linear approximation for $u'\left(c\right)$ to derive

$$h\left(a\right) \approx \frac{a^\rho - 1 + \sigma^2}{\left(a^\rho + 1\right)\sigma^2 - \left(a^\rho - 1\right)^2}.$$

you might also derive

$$h\left(a\right) \approx \frac{1}{2} + \frac{a^\rho - 1}{2\sigma^2}$$

(it depends wether you linearize only w.r.t. ϵ and ϵ^*, or also w.r.t. a.)

Use these relations to argue that the derivative $h'\left(1\right)$ is larger for smaller values of σ^2 and for larger values of ρ. Give an economic interpretation.

(v) Suppose the outside world does not invest in the country. Show that the current account can be written as

$$CA_t = \left(1-\theta_t\right)\left(w_t - c_t\right) - \left(1-\theta_{t-1}\right)\left(w_{t-1} - c_{t-1}\right).$$

Derive the usual current account identity for this economy.

(vi) Show that the effect of a productivity shock on CA_t can be decomposed in two components, one due to Δw_t one due to $\Delta \theta_t$. Suppose we are in a situation where $A_{t-1} = A^*_{t-1} = 1$. Show that if h' is small the first component dominates and a domestic productivity shock has a positive effect on the current account surplus, if h' is large the second component dominates and a productivity shock has a negative effect on the current account surplus. Use (iii) and (iv) to interpret this result.

3. (Rogoff, 1992)

Consider a small open economy with a representative consumer with preferences

$$\mathrm{E} \sum \beta^t U(c_t)$$

where $U(c) = c^{1-\gamma}/(1-\gamma)$ and where consumption is the following aggregate of tradable and non-tradable goods

$$c_t = \left(c_t^T\right)^{\alpha} \left(c_t^N\right)^{1-\alpha}.$$

Production functions of tradables and non-tradables are Cobb-Douglas:

$$\begin{aligned} y_t^T &= A_t^T \left(n_t^T\right)^{\alpha}, \\ y_t^N &= A_t^N \left(n_t^N\right)^{\alpha}. \end{aligned}$$

The consumer can only trade a one-period non-state-contingent bond. There is no capital. The world interest rate is fixed at r. Consumers have a unit supply of labor which is fully employed in one of the two sectors:

$$n_t^T + n_t^N = 1.$$

(i) Write the budget constraint and derive the optimality conditions for the consumer and for the firms producing tradables and non-tradables. Use tradables as the numeraire.

(ii) Using consumer optimality derive a relation between the ratio c_t^T/c_t^N and the relative price of non-tradables p_t.

(iii) Using firms' optimality and labor market clearing derive a relation between n_t^T, the relative price of non-tradables and the productivity levels A_t^T and A_t^N.

(iv) Combine (ii) and (iii), the production function for non-tradables and market clearing in non-tradables to find a relation that must hold each period between c_t^T and n_t^T.

Suppose the country has constant productivities $A_t^T = A_t^N = \bar{A}$. At date 0 the country experiences a one-time, unexpected, positive transitory shock to the productivity of tradables so

$$A_t^T = \bar{A} + \rho^t \epsilon$$

with $\epsilon > 0$ and $\rho \in (0, 1)$.

(v) Assume $\gamma = 1$. What is the effect of the productivity shock on consumption of tradables c_t^T at dates $0, 1, 2, ...$?

(vi) What is the effect on the production and consumption of non-tradables (hint: use your result in part (iv))?

(vii) What is the effect on the relative price p_t at dates $0, 1, 2, ...$? Does it depend on ρ? Does this support Rogoff's claim that "barring shocks to the supply of non-traded goods available for private consumption, the log real exchange rate would follow a random walk, regardless of the serial correlation properties of the shocks to traded goods productivity (p. 12 of NBER WP 4119).